MTP-85

PERFORMANCE OF TWO-DIMENSIONAL ERROR DETECTION ON DIGITAL HF AND TROPOSCATTER CHANNELS

K. Brayer

DECEMBER 1968

Prepared for

AEROSPACE INSTRUMENTATION PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 705B

Prepared by

THE MITRE CORPORATION
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FOREWORD

This report was prepared by The Communications Techniques Department of The MITRE Corporation, Bedford, Massachusetts, under Contract AF 19(628)-5165. The work was directed by the Ground Instrumentation Engineering Division under the Aerospace Instrumentation Program Office, Air Force Electronic Systems Division, Laurence G. Hanscom Field, Bedford, Massachusetts. Robert E. Forney served as the Air Force Project Engineer for this program, identifiable as ESD (ESSIC) Project 5932, Range Data Transmission.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

GEORGE T. GALT, Colonel, USAF
Director of Aerospace Instrumentation
Program Office

ABSTRACT

The problem of applying horizontal and/or vertical parity checks for error detection to actual burst channels is considered. It is demonstrated that a single-dimension parity check will achieve two to four orders-of-magnitude improvement, and, within the limit of the data sample, both dimensions together detect all errors.

It is proved that the parity check at the intersection of the horizontal and vertical checks can be calculated from either set of checks and that errors can be detected in the information independent of those in the non-intersecting checks.

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PERFORMANCE OF TWO-DIMENSIONAL ERROR DETECTION ON DIGITAL HF AND TROPOSCATTER CHANNELS

INTRODUCTION

In previous work Kuhn[1] analyzed the error-detection performance of two-dimensional (horizontal/vertical) binary parity check codes in a random-error channel and predicted the undetected block error probabilities. Owing to a lack of burst statistics, Kuhn gave only a qualitative analysis of performance in real burst channels. In this paper the error-detection performance of this scheme shall be described for the previously reported high-frequency[2] and troposcatter[3] channels in which errors occur in bursts.

STRUCTURE OF THE CODE

A two-dimensional binary parity check code is structured by taking a sequence of m binary digits from the information source and calculating a parity bit. This generates an (m + 1)-bit word. The process is repeated n times and to the n words is appended a parity word. The parity word is

calculated by using the first bit of each information word to find the first bit of the parity word and repeating the calculation for the m information bits/word. As proved in the Appendix, the last bit of the parity word can be calculated from the word parity bits or from the bits of the parity word. Because of the relationship of the bits in the parity word to the information bits, the parity word is frequently called vertical or column parity. Similarly, the word parity bits are frequently called horizontal or row parity. The parity bits are calculated under an even-calculation rule. This rule requires that the sum of all bits in a row (column) be even, where addition is modulo 2 with no carry and zero is an even number. Thus, in encoding, if the sum of the information bits in a row (column) is odd the parity bit is a "one." In decoding, each row (column) is checked to see if the sum is still even (parity is satisfied). If it is not, an error is declared as detected. It is possible, however, that an even number of errors will occur in a row (column). In this case the sum is not changed and the errors are not detected in the row (column). Further, if odd parity were originally used in the encoding and decoding rules the performance would be the same. The implementation of this code requires $m + 1$ flip-flops. Of these, m are used for the information column calculations, and since rows are processed sequentially, only one is needed for the row calculations including the calculation on the parity word.

REVIEW OF CHANNEL DATA

In October of 1965, field tests were conducted on NRD high-frequency circuits between Antigua and Ascension Islands to record HF digital error patterns[2]. Tests were conducted for 10-minute periods (runs) at 2400 bits/sec using Kineplex TE-216 16-tone PSK modems. A 52-bit test message was transmitted from Antigua to Ascension and back to Antigua on a looped

basis, where the received message was compared with a suitably delayed replica of the test message. Bits not in agreement were declared in error and recorded for computer processing. The troposcatter data[3] were collected using Sebit 24B vestigial sideband AM modems at 2400 bits/sec on a link dominated by a 583-mile troposcatter hop. Test runs were 90 minutes long. The data are summarized in Table I.

Table I

Size of Data Sample

Channel	Total Bits	Bit Error Rate $= \dfrac{\text{Total Errors}}{\text{Total Bits}}$
High-Frequency	4.89×10^{7}	5.47×10^{-3}
Troposcatter	5.26×10^{8}	3.95×10^{-4}

Analysis of the HF data[2] indicates that the errors occur in bursts of approximately 5% error density and 3000-bit average length. Troposcatter bursts[3] occur in densities ranging from 10% to a few of 75% and average less than 1000 bits in length.

PERFORMANCE RESULTS

For purposes of analysis it is assumed that an all-zero message is transmitted. This message satisfies even parity. If the error patterns are added to the message sequence the result is the error-pattern data. Thus the code performance is evaluated by checking the error patterns for even parity. The errors were divided into blocks of $(n + 1)$ rows and $(m + 1)$ columns containing n rows and m columns of information. Within each

block having errors, rows and columns were checked for parity. If any parity check was not satisfied, a block in error was declared as detected. Since the number of blocks with errors was also known, the undetected error rate could be obtained. Results were also obtained using row or column parity check detection only.

As an example, the results for $m = 8$ and $n = 7$ on the tropospheric data are as shown in Table II.

Table II

Performance of Horizontal/Vertical Parity Check

Block Size	9 columns x 8 rows = 72 bits
Total Blocks	7,309,865
Blocks in Error	27,181
Case A: Blocks Detectable Using Row Parity Only	25,332
Case B: Blocks Detectable Using Column Parity Only	26,984
Case C: Blocks Detectable Using Two-Dimensional Parity	27,177
Channel Block Error Rate	.0037
Undetected Block Error Rate:	
Case A	.00025
Case B	.000026
Case C	.00000055

From the table the burst nature of the channel is evident in that only 27,181 of 7.3 million blocks had errors. Since data are transmitted

sequentially in words, the bursts occur across rows, leading to poorer row undetected error rates. In Figures 1 and 2 the results are presented for both data channels for the cases of transmission of 8-bit ($m = 8$) computer bytes and square blocks, respectively. For the HF data there is always a block length beyond which there are no undetected block errors, and for $m = 8$ most rows have but one error, thus favoring row detection over column detection. In transmission of computer bytes through the troposcatter channel there are four patterns of errors which sometimes go undetected, but square blocks are always detectable on a block basis. Had the messages been interleaved [4] prior to transmission, the square block row and column performance results would have been interchanged.

CONCLUSIONS

Within the size of the data sample used (6 hrs HF and 61 hrs troposcatter) the limits of performance have not yet been reached. One can only speculate on how much data need be collected before a valid measure of undetected error probability is attained, but in any event it appears that a test program to collect such data would be impractically long and expensive. The results cannot be compared to those of Kuhn[1], since to validate undetected error rates of 10^{-14} would require 60,000,000 hrs of data at 2400 bits/sec. It can be concluded, however, that simple parity gives excellent error-detection performance at a reasonable cost. Furthermore, two dimensions of error detection are far superior to either horizontal or vertical alone. Additionally, the use of this simple scheme gives lower undetected error rates than those previously reported[5] for BCH codes.

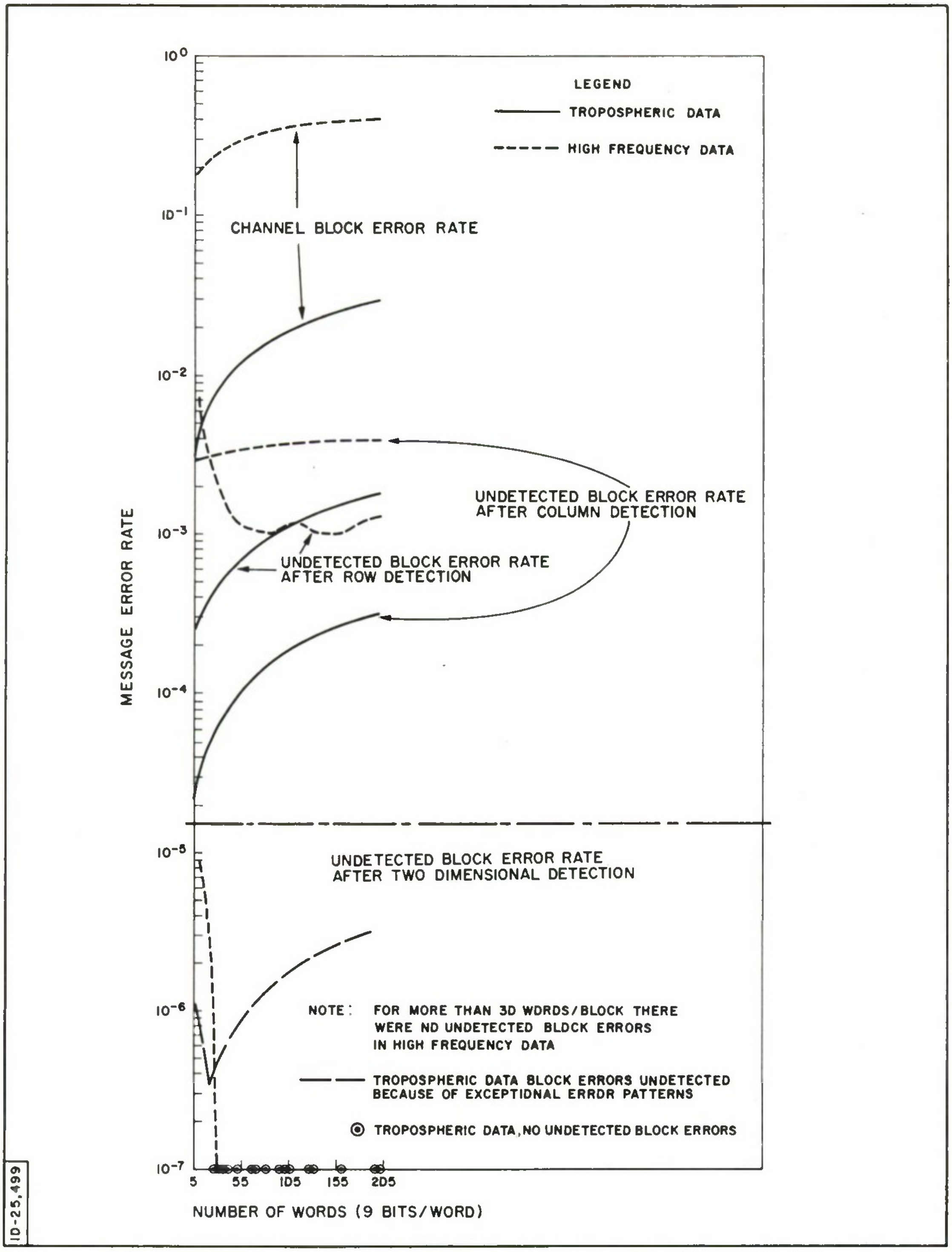

Figure 1. Performance of Two-Dimensional Parity Check Error Detection (Computer Bytes)

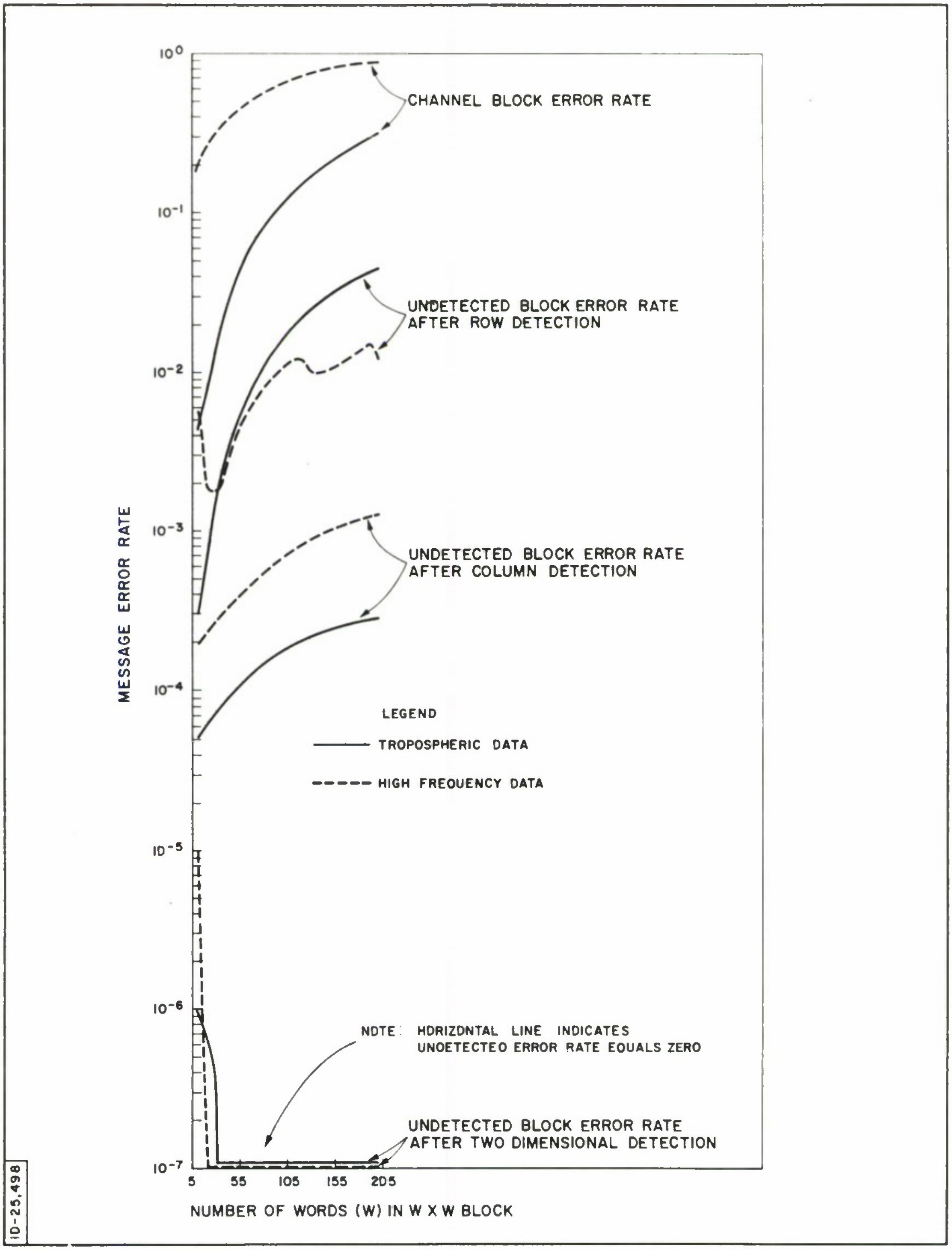

Figure 2. Performance of Two-Dimensional Parity Check Error Detection (Square Block)

APPENDIX

In this appendix a theorem which can be used in encoding a two-dimensional parity check matrix is proved. It is demonstrated that the encoding rule must be modified if the dimensions of the matrix are not the same in kind (odd or even) and odd parity is used. A corollary to this theorem permits the detection of errors in transmitted information independent of the errors in the one-dimensional parity bits.

Theorem:

In a two-dimensional parity check matrix, with n information words and m information bits/word, where each word has a parity bit appended to it and a parity word is appended to the matrix; the bit of the parity word which corresponds to the word parity bits can be calculated from either set of parity bits.

Proof (Case 1 - Even Parity Rule):

Given a matrix of information bits A with elements $a_{i,j}$ $(a_{i,j} = 0, 1)$, $(i = 1, 2, \ldots, n)$, $(j = 1, 2, \ldots, m)$. With an even parity constraint, the word parity bit $a_{i,m+1}$ is given by

$$a_{i,m+1} = \sum_{j=1}^{m} a_{i,j} \qquad [\text{modulo } 2] \ , \ i = 1, 2, \ldots, n \ . \qquad (1)$$

The block parity bit when calculated from the word parity bits is given by

$$a_{n+1,m+1} = \sum_{i=1}^{n} a_{i,m+1} \qquad [\text{modulo } 2] \tag{2}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j} \qquad [\text{modulo } 2] \; . \tag{3}$$

If the sum is reordered,

$$a_{n+1,m+1} = \sum_{j=1}^{m} a_{n+1,j} \qquad [\text{modulo } 2] \tag{4}$$

where $a_{n+1,j}$ is given by

$$a_{n+1,j} = \sum_{i=1}^{n} a_{i,j} \qquad [\text{modulo } 2] \; , \; j = 1,2,\ldots,m \; . \tag{5}$$

Comparing Equations (2) and (4) it is found that while the bit $a_{n+1,m+1}$ was originally calculated from the word parity bits the result is the same as if it had been found from the bits of the parity word.

<u>Proof (Case 2 - Odd Parity Rule):</u>

Sub-case 2a (m and n are even).

In sub-case 2a, Equation (1) is replaced by

$$a_{i,m+1} = 1 + \sum_{j=1}^{m} a_{i,j} \qquad [\text{modulo } 2] \; , \; i = 1,2,\ldots n \; . \tag{6}$$

It follows that

$$a_{n+1,m+1} = 1 + \sum_{i=1}^{n} a_{i,m+1} \qquad \text{[modulo 2]} \tag{7}$$

$$= 1 + \sum_{i=1}^{n} \left(1 + \sum_{j=1}^{m} a_{i,j}\right) \qquad \text{[modulo 2]} \tag{8}$$

$$= 1 + \sum_{i=1}^{n} (1) + \sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j} \qquad \text{[modulo 2]} \tag{9}$$

$$= 1 + \sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j} \qquad \text{[modulo 2]} \tag{10}$$

$$= 1 + \sum_{j=1}^{m} a_{n+1,j} \qquad \text{[modulo 2]} \tag{11}$$

where

$$a_{n+1,j} = 1 + \sum_{i=1}^{n} a_{i,j} \qquad \text{[modulo 2]}\ ,\ j = 1, 2, \ldots m. \tag{12}$$

Sub-case 2b (m and n are odd).

In this sub-case, the derivation follows that of sub-case 2a with the exception that the value $a_{n+1,m+1}$ equals one plus that value of sub-case 2a.

Sub-case 2c (m and n are different in kind).

In sub-case 2c, calculation of $a_{n+1,m+1}$ yields two answers differing by one depending on the bits used in the calculation. In this sub-case, the calculation using an even number of terms must be increased by one.

This proof can be extended to an r-dimensional parity check matrix, but such matrices are not used in practice.

The above theorem leads to the following corollary.

Corollary:

Parity bit $a_{n+1, m+1}$ detects an odd number of errors in the information portion of the block (including bit $a_{n+1, m+1}$) independent of errors in the other parity bits.

This is a direct result of Equations (3) and (10).

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